

CANONICAL CONTACT STRUCTURES ON FIBRED SINGULARITY LINKS

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ABSTRACT. We identify the canonical contact structure on any singularity link which fibers over the circle by drawing a Legendrian handlebody diagram of one of its Stein fillings. We also prove that the Euler class of such a contact structure vanishes although the Stein filling we construct has non-vanishing first Chern class.

1. INTRODUCTION

There is a canonical (a.k.a. Milnor fillable) contact structure on the link of an isolated complex surface singularity, which is unique up to isomorphism [4]. In general it is a difficult task to identify this canonical contact structure [16]. In this article we determine the canonical contact structure on any singularity link which fibers over the circle by drawing a Legendrian handlebody diagram (cf. [10, 11]) of one of its Stein fillings. Moreover we convert this diagram into a surgery diagram of the canonical contact structure by replacing any 1-handle in the Stein filling by a contact (+1)-surgery along a Legendrian unknot in the standard contact S^3 (cf. [5, 6]). Furthermore we prove that the Euler class of the canonical contact structure on any singularity link which fibers over the circle vanishes although the Stein filling we construct has non-vanishing first Chern class.

The discussion in this article is based on the topological characterization of singularity links which fiber over the circle given by Neumann [17]: A singularity link fibers over the circle if and only if it is a torus bundle over the circle whose monodromy $A \in SL(2, \mathbb{Z})$ is either parabolic, i.e., $tr(A) = 2$ or hyperbolic with $tr(A) \geq 3$.

It is known that a Milnor fillable contact structure is Stein fillable [3] and universally tight [14]. Notice that a Stein fillable contact structure is not necessarily universally tight, i.e., it may have overtwisted finite covers (see Gompf's examples in [10]).

On the other hand, the classification of tight contact structures on torus bundles by Honda [12] coupled with a theorem of Gay [9] implies that on any torus bundle over the circle there is a unique Stein fillable universally tight contact structure up to isomorphism. We conclude that on a singularity link which fibers over the circle, the canonical contact structure is the unique universally tight Stein fillable contact structure.

In the following we divide the proof of our results into parabolic and hyperbolic cases. All the contact structures are assumed to be positive and co-orientable.

2. PARABOLIC CASE

A torus bundle over the circle with parabolic monodromy $A \in SL(2, \mathbb{Z})$ also admits a circle fibration over T^2 with *negative* Euler number $-n$ for some positive integer n . Let Y_n denote the total space of such a bundle. An open book decomposition \mathcal{OB}_n of Y_n transverse to the circle fibration was constructed in [7] such that the binding consists of n distinct circle fibres, page is a torus with n boundary components and monodromy is a product of one right-handed Dehn twist about a curve parallel to each boundary component. By Theorem 2.1 of [18], there is a Milnor open book $\widetilde{\mathcal{OB}}_n$ on Y_n whose binding agrees with the binding of \mathcal{OB}_n . On the other hand, by [4, Theorem 4.6], any two horizontal open books on Y_n with the same binding are isomorphic. It follows that \mathcal{OB}_n is in fact a Milnor open book and hence it supports the canonical contact structure which is certainly Stein fillable and universally tight. Let ξ_{can} denote the *canonical* contact structure on the singularity link Y_n .

Lemma 1. *The Poincaré dual of the Euler class $e(\xi_{can})$ vanishes in the first integral homology group of Y_n .*

Proof. From a given open book, one can compute the Poincaré dual of the Euler class of the compatible contact structure by the algorithm described in [8]. In the case at hand, ξ_{can} on Y_n is compatible with \mathcal{OB}_n which is explicitly constructed above. We assume that $n \geq 2$, since the result follows directly from [8, Lemma 6.1] for $n = 1$. First we need to calculate the rotation numbers (on a page) of all the curves which appear in the factorization of the monodromy of \mathcal{OB}_n into a product of Dehn twists. We depicted all the boundary parallel curves on a page in Figure 1. It follows that rotation number $\text{rot}(\gamma_i) = 0$ for $2 \leq i \leq n$ and $\text{rot}(\gamma_1) = \pm n$ where the sign depends on the chosen orientation.

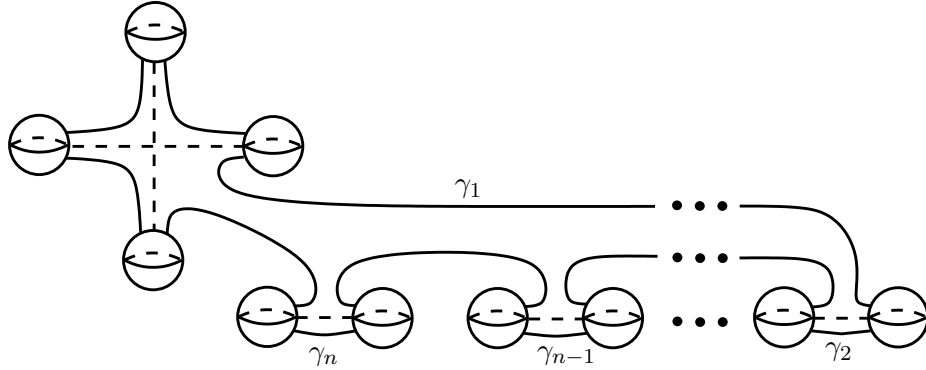


FIGURE 1. Boundary parallel curves $\gamma_1, \gamma_2, \dots, \gamma_n$ on a page of the open book \mathcal{OB}_n of Y_n .

Next we observe that by assigning -1 framing to the curves $\gamma_1, \dots, \gamma_n$ in Figure 1, we obtain a smooth handlebody diagram of a Stein filling of Y_n (cf. [1]). Moreover, a surgery diagram of the 3-manifold Y_n can be obtained by sliding the 2-handle γ_1 over the 2-handles $\gamma_2, \dots, \gamma_n$, canceling the 2-handle γ_i with the 1-handle carrying γ_i for $i = 2, \dots, n$, and finally converting the 1-handles into zero framed circles as shown in Figure 2. Notice that the curve γ_1 in Figure 1 corresponds to the $-n$ framed curve in Figure 2. Furthermore, the homology class represented by the meridional circle μ_1 is the generator of the torsion \mathbb{Z}_n , while the homology classes represented by μ_a and μ_b generate the free part $\mathbb{Z} \oplus \mathbb{Z}$ in $H_1(Y_n; \mathbb{Z})$. Therefore we calculate that

$$PD(e(\xi_{can})) = \pm n[\mu_1] = 0 \in H_1(Y_n; \mathbb{Z}) = \mathbb{Z}_n \oplus \mathbb{Z} \oplus \mathbb{Z}.$$

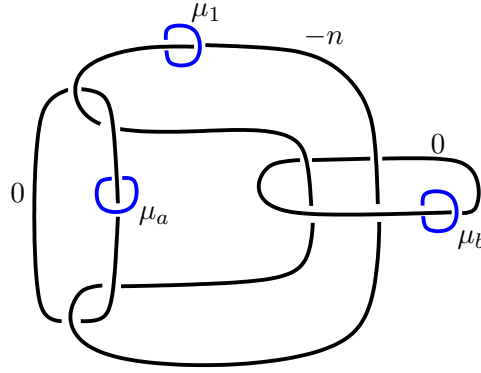


FIGURE 2. Surgery diagram for Y_n , where the homology classes of the meridional circles μ_a, μ_b and μ_1 generate $H_1(Y_n; \mathbb{Z})$.

□

Remark 2. Let (S_n, p) denote the singularity whose minimal resolution consists of a single elliptic curve of negative self-intersection number $-n$ for some $n \in \mathbb{N}$. Such singularities are known as simple elliptic singularities. Since simple elliptic singularities are Gorenstein, and by a result of Seade [21], the canonical class of any smoothing of S_n is trivial. Combining this with a result of Pinkham [20], which states that S_n admits smoothing if and only if $n \leq 9$, we see that the Euler class of the canonical contact structure on Y_n vanishes if $n \leq 9$. Lemma 1 shows that the Euler class of the canonical contact structure on Y_n vanishes in the case $n > 9$ as well.

Proposition 3. There are precisely two distinct isotopy classes of Stein fillable universally tight contact structures on Y_n . These are given as the boundaries of the two distinct Stein

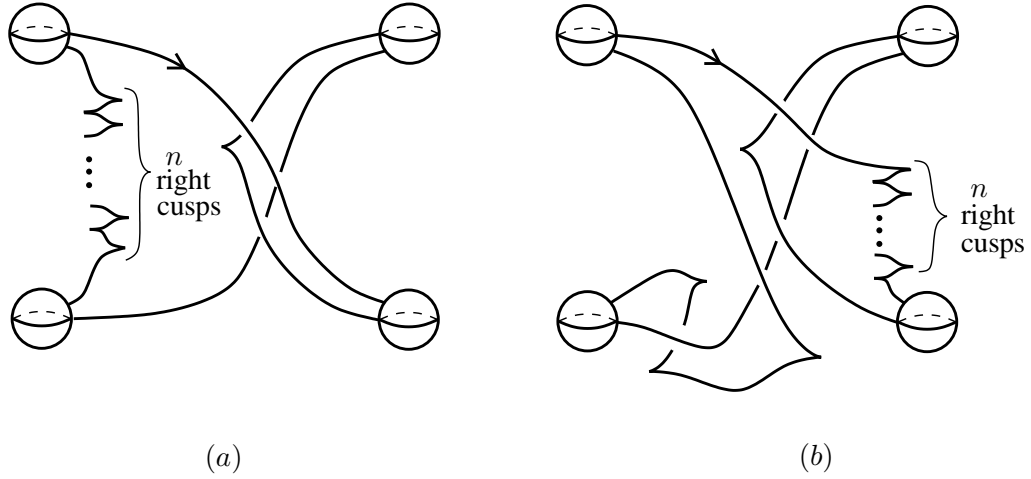


FIGURE 3. Legendrian handlebody diagrams for two distinct Stein structures on X_n , both inducing the canonical contact structure ξ_{can} on Y_n , up to isomorphism.

surfaces shown in Figure 3. Both of these contact structures represent the isomorphism class of the canonical contact structure ξ_{can} on Y_n .

Proof. There are $n + 1$ distinct isotopy classes of Stein fillable contact structures on Y_n , two of which are universally tight according to Honda's classification [12]. Let X_n denote the D^2 -bundle over T^2 with Euler number $-n$. The distinct Stein fillable contact structures on Y_n can be described as the boundaries of distinct Stein structures on X_n . To obtain distinct Stein structures on X_n , one can convert the standard smooth handlebody diagram of X_n (which consists of two 1-handles and one 2-handle) into a Legendrian handlebody diagram by Legendrian realizing the attaching circle of the 2-handle [10]. The first Chern class of such a Stein structure is represented by a cocycle whose value on the homology class induced by the oriented Legendrian knot (generating $H_2(X_n; \mathbb{Z}) \cong \mathbb{Z}$) is given by the rotation number of this Legendrian knot. Therefore the induced contact structures on Y_n are nonisotopic for any distinct rotation numbers realized by the oriented Legendrian knot (cf. [15]). Moreover, by [10, Proposition 5.1], all except two that are depicted in Figure 3 are virtually overtwisted. The rotation numbers of the Legendrian knots in Figure 3 (a) and Figure 3 (b) are $-n$ and n , respectively, with the indicated orientations. Here if one reverses the orientation of the Legendrian knot, the sign of the rotation number as well as the sign of the second homology class induced by this knot in $H_2(X_n; \mathbb{Z})$ gets reversed. In other words, the two extreme cases where the rotation number takes its minimal possible value $-n$ and maximum possible value n , respectively, induce the two universally tight Stein fillable contact structures on Y_n in Honda's classification.

To prove the last statement in the proposition, we first observe that one of two nonisotopic Stein fillable universally tight contact structures on Y_n is ξ_{can} . Let $\bar{\xi}_{can}$ denote the 2-plane field ξ_{can} with the opposite orientation. We claim that $\bar{\xi}_{can}$ represents the other such contact structure on Y_n and is isomorphic to ξ_{can} . To see this recall that ξ_{can} is supported by the Milnor open book \mathcal{OB}_n . By reversing the orientation of the page (and hence the orientation of the binding) of \mathcal{OB}_n we get another elliptic open book $\overline{\mathcal{OB}}_n$ on Y_n . The open book $\overline{\mathcal{OB}}_n$ is in fact isomorphic to \mathcal{OB}_n , since they have identical pages and the same monodromy map measured with the respective orientations. To see that $\overline{\mathcal{OB}}_n$ is also horizontal we simply reverse the orientation of the fibre (to agree with the orientation of the binding) as well as the orientation of base T^2 of the circle bundle Y_n , so that we do not change the orientation of Y_n . In addition we observe that the contact structure supported by $\overline{\mathcal{OB}}_n$ can be obtained from ξ_{can} by changing the orientations of the contact planes since $\overline{\mathcal{OB}}_n$ is obtained from \mathcal{OB}_n by changing the orientations of the pages. This immediately implies that $\bar{\xi}_{can}$ is isomorphic to ξ_{can} . \square

Notice that we can trade the 1-handles in the handlebody diagram depicted in Figure 3 with contact $(+1)$ -surgeries as described in [6] and obtain the contact surgery diagram of ξ_{can} on Y_n as shown in Figure 4.

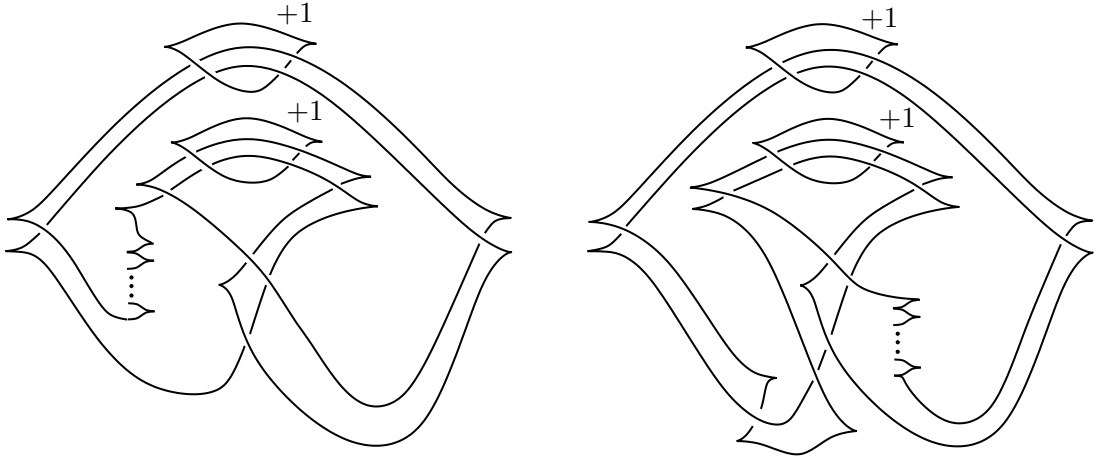


FIGURE 4. Two contact surgery diagrams inducing non-isotopic contact structures isomorphic to ξ_{can} on Y_n .

We would like to point out that the first Chern classes of both Stein fillings of the contact 3-manifold (Y_n, ξ_{can}) depicted in Figure 3 are non-vanishing, although their restriction to the boundary vanishes.

Remark 4. *Ohta and Ono [19] have shown that (Y_n, ξ_{can}) admits a strong symplectic filling with vanishing first Chern class if and only if $n \leq 9$. Moreover they proved that any such filling is diffeomorphic to a smoothing of the singularity, which is unique unless $n = 8$. Stein fillings with vanishing first Chern class can be constructed as a PALF (positive allowable Lefschetz fibration [1]) using the n -holed torus relations discovered in [13]. Unless $n = 8$, such a Stein filling represents the unique diffeomorphism type mentioned above.*

3. HYPERBOLIC CASE

For the hyperbolic case (i.e., when $A \in SL(2, \mathbb{Z})$ with $tr(A) \geq 3$) we first observe that in Neumann's paper [17], there is a circular plumbing graph of the torus bundle depending on a factorization of the monodromy $A = A(n_1, \dots, n_k)$. Here the Euler number of the i -th vertex is given by $-n_i$ where $n_i \geq 2$ for all i , and $n_i \geq 3$ for some i . To simplify the notation, we denote the total space by $Y_{\bar{n}}$, where $\bar{n} = (n_1, \dots, n_k)$, and the circular plumbing graph by $\Gamma_{\bar{n}}$ as shown in Figure 5.

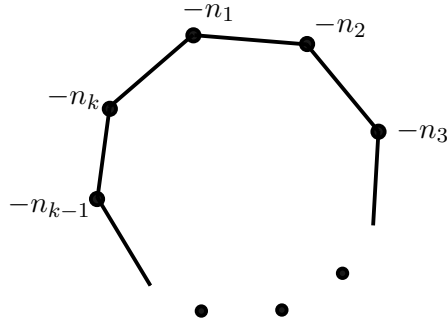
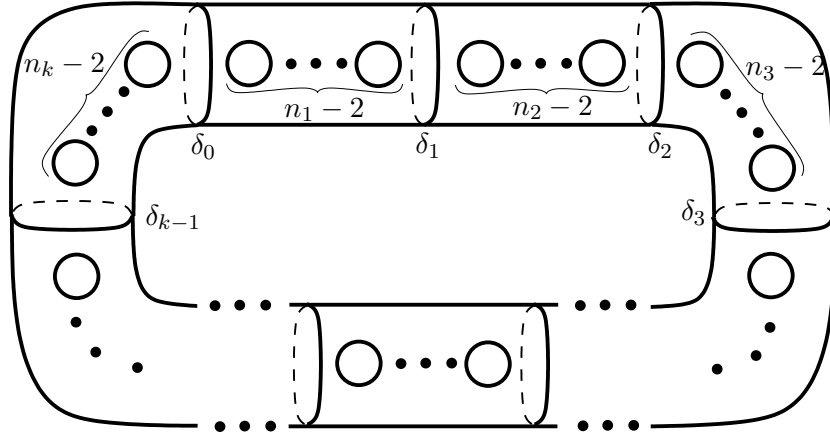


FIGURE 5. The circular plumbing graph $\Gamma_{\bar{n}}$ for the singularity link $Y_{\bar{n}}$. The vertex labeled by $-n_i$ represents a circle bundle over S^2 with Euler number $-n_i$.

Lemma 5. *There is an elliptic open book decomposition $\mathcal{OB}_{\bar{n}}$ on $Y_{\bar{n}}$ such that the monodromy is a product of right-handed Dehn twists along boundary parallel curves and some meridional curves.*

Proof. We will construct an elliptic open book $\mathcal{OB}_{\bar{n}}$ using the methods in [7]. Notice that the i -th vertex in the plumbing is a circle bundle over the sphere with Euler number $-n_i \leq -2$. For a such a circle bundle there is an open book \mathcal{OB}_i where the page is a sphere with n_i boundary components and monodromy is a product of one right-handed Dehn twist along a curve parallel to each boundary component.

FIGURE 6. The page of the open book $\mathcal{OB}_{\overline{n}}$ of $Y_{\overline{n}}$

When two consecutive circle bundles with Euler numbers $-n_i$ and $-n_{i+1}$ are connected by an edge in the plumbing graph $\Gamma_{\overline{n}}$, we can “glue” the corresponding open books together as follows: First of all, for $i = 1, \dots, k-1$, we glue a page of \mathcal{OB}_i with a page of \mathcal{OB}_{i+1} using precisely one boundary component from each page. The Dehn twists along the identified boundary components merge into a single right-handed Dehn twist along the resulting meridional curve δ_i (see Figure 6) after gluing the pages. By gluing all the open books corresponding to the vertices $i = 1, \dots, k-1$, we get a planar open book. But since the plumbing graph is circular we need to glue the \mathcal{OB}_k with \mathcal{OB}_1 along δ_0 so that the resulting page of the open book $\mathcal{OB}_{\overline{n}}$ is a torus with $\sum_{i=1}^k n_i - 2$ many boundary components as illustrated in Figure 6. Let $\gamma_{i,1}, \gamma_{i,2}, \dots, \gamma_{i,n_i-2}$ be the boundary parallel curves between δ_{i-1} and δ_i for $1 \leq i \leq k-1$, and let $\gamma_{k,1}, \gamma_{k,2}, \dots, \gamma_{k,n_k-2}$ be the boundary parallel curves between δ_{k-1} and δ_0 . Then the monodromy of $\mathcal{OB}_{\overline{n}}$ is given by

$$\prod_{i=0}^{k-1} D(\delta_i) \prod_{i=1}^k \prod_{j=1}^{n_i-2} D(\gamma_{i,j}),$$

where $D(\cdot)$ denotes a right-handed Dehn twist.

□

Remark 6. *The open books we constructed on torus bundles coincides with the ones given in [22, Chapter 4] although our method is completely different.*

Lemma 7. *The open book $\mathcal{OB}_{\overline{n}}$ on $Y_{\overline{n}}$ is a Milnor open book.*

Proof. According to [18, Theorem 2.1], there is an analytic structure (Z, p) on the cone over $Y_{\overline{n}}$ and a corresponding Milnor open book $\widetilde{\mathcal{OB}}_{\overline{n}}$ on $Y_{\overline{n}}$ whose binding agrees with the

binding of $\mathcal{OB}_{\overline{n}}$. Now the circular plumbing graph $\Gamma_{\overline{n}}$ provides a decomposition of $Y_{\overline{n}}$ into a union $\bigcup V_i$, where V_i is an S^1 -bundle over S^2 with 2 discs removed. Since any page of $\mathcal{OB}_{\overline{n}}$ intersects in V_i in exactly one component for each i , by the argument of the proof of Theorem 4.6 of [4], any horizontal open book whose binding agrees with the binding of $\mathcal{OB}_{\overline{n}}$ must be isomorphic to $\mathcal{OB}_{\overline{n}}$. Thus $\mathcal{OB}_{\overline{n}}$ is indeed a Milnor open book. \square

Lemma 8. *The Poincaré dual of the Euler class $e(\xi_{can})$ vanishes in the first integral homology group of $Y_{\overline{n}}$.*

Proof. The proof of this result is analogous to the proof of Lemma 1. We may assume that $n_1 > 2$, by renaming the vertices of the plumbing graph $\Gamma_{\overline{n}}$ if necessary. We calculate that the rotation number $\text{rot}(\delta_{k-1}) = \pm(n_k - 2)$ as shown in Figure 7, where the sign depends on the orientation.

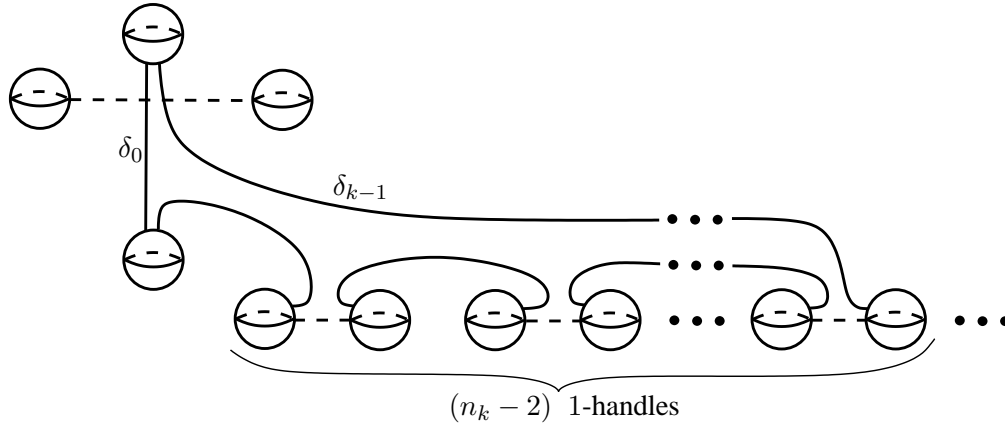


FIGURE 7. The curves δ_0, δ_{k-1} are depicted on the page of the open book $\mathcal{OB}_{\overline{n}}$. To draw the rest of the page one first adds $(n_{k-1} - 2)$ more 1-handles to the right-side, and slide δ_{k-1} over these as well to obtain δ_{k-2} , etc.

We fix an orientation of the curve δ_0 and orient the curves $\delta_1, \dots, \delta_{k-1}$ accordingly so that $\text{rot}(\delta_{k-1}) = n_k - 2$. Then we calculate that

$$\text{rot}(\delta_i) = \sum_{j=i+1}^k n_j - 2$$

for $1 \leq i \leq k-1$ and $\text{rot}(\delta_0) = 0$. Notice that all the boundary curves except for $\gamma_{1,1}$ has zero rotation number. We calculate that $\text{rot}(\gamma_{1,1}) = \sum_{i=1}^k n_i - 2$.

Next we find a surgery presentation of the 3-manifold $Y_{\overline{n}}$. By assigning -1 framing to all the boundary curves and to the curves $\delta_0, \delta_1, \dots, \delta_{k-1}$ in Figure 7, we get a smooth

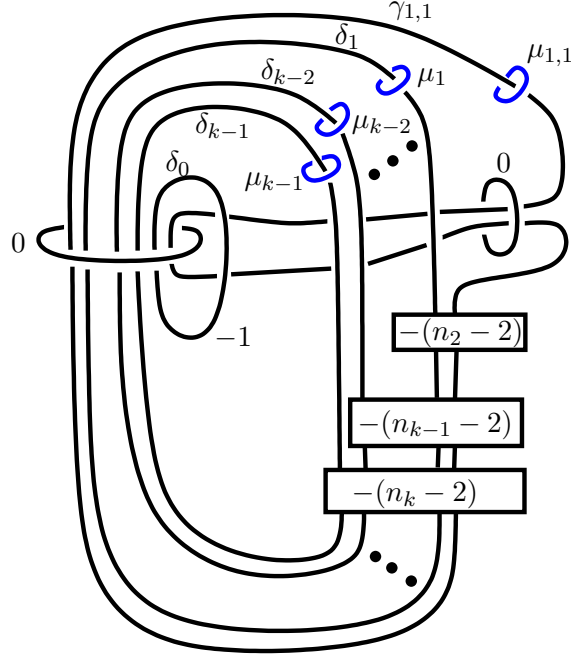


FIGURE 8. Surgery diagram for $Y_{\overline{n}}$, where $\gamma_{1,1}$ has framing $-\text{rot}(\gamma_{1,1})$, and δ_i has framing $-\text{rot}(\delta_i)$, for $1 \leq i \leq k-1$.

handlebody diagram of a Stein filling of $Y_{\overline{n}}$. By performing several handle slides and canceling all the 1-handles except for two, and converting them to zero framed circles we obtain the diagram in Figure 8.

In Figure 8, we indicated representatives of some of the homology generators of $H_1(Y_{\overline{n}}; \mathbb{Z})$ by the meridians μ_1, \dots, μ_{k-1} and $\mu_{1,1}$. Notice that the curve $\gamma_{1,1}$ has framing $-\text{rot}(\gamma_{1,1})$ and the linking number $\text{lk}(\gamma_{1,1}, \delta_i) = -\text{rot}(\delta_i)$, for $1 \leq i \leq k-1$. Therefore we conclude that

$$PD(e(\xi_{can})) = \text{rot}(\gamma_{1,1})\mu_{1,1} + \sum_{i=1}^{k-1} \text{rot}(\delta_i)\mu_i = 0 \in H_1(Y_{\overline{n}}; \mathbb{Z}).$$

□

Proposition 9. *There are precisely two distinct isotopy classes of Stein fillable universally tight contact structures on $Y_{\overline{n}}$. One of them is the contact structure induced on the boundary of the Stein surface depicted in Figure 9 where each Legendrian 2-handle attains its minimal possible rotation number. The other one is obtained, similarly, by achieving the maximal possible rotation number for each Legendrian 2-handle. Both of these contact*

structures represent the isomorphism class of the canonical contact structure ξ_{can} on $Y_{\vec{n}}$.

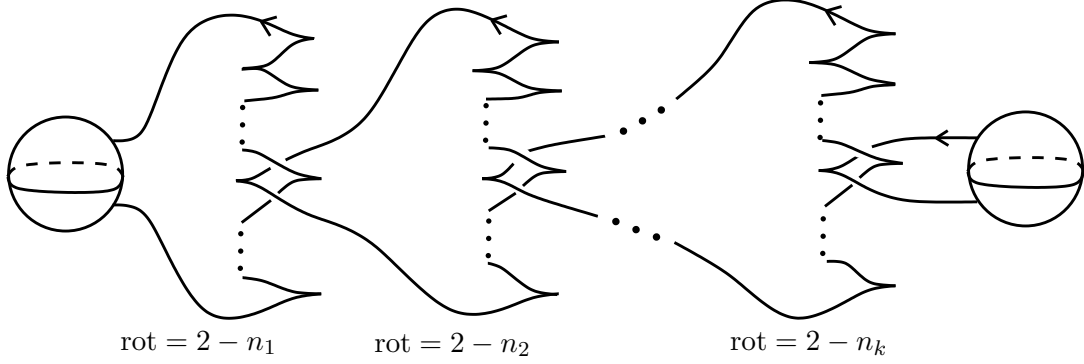


FIGURE 9. Legendrian handlebody diagram of a Stein filling of the contact 3-manifold $(Y_{\vec{n}}, \xi_{can})$.

Proof. On the torus bundle $Y_{\vec{n}}$, there are $(n_1 - 1)(n_2 - 1) \cdots (n_k - 1)$ distinct isotopy classes of Stein fillable contact structures two of which are universally tight according to Honda's classification [12]. These contact structures can be obtained as follows: Consider the simple surgery description of the 3-manifold $Y_{\vec{n}}$ which we depicted in Figure 10 (see [17]).

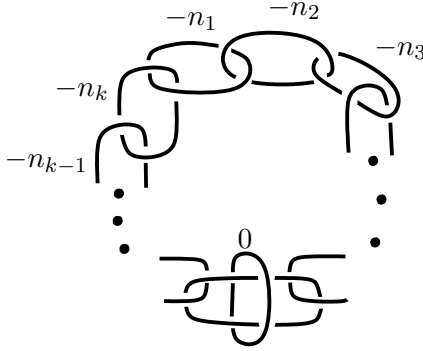


FIGURE 10. Surgery description of $Y_{\vec{n}}$

To realize any of the contact structures above, first convert the 0-framed unknot linking the chain into a 1-handle such that only one of the 2-handles goes over the 1-handle twice with zero linking. Then all the unknots not going over the 1-handle with framings ≤ -2 can be Legendrian realized as usual. So, one can produce all of the finite number

of Stein fillable contact structures above by considering all possible rotation numbers for each Legendrian unknot in this diagram. We claim that the canonical one must be the one where we minimize the rotation numbers of all the knots as in Figure 9 (or the one where we maximize the rotation numbers of all the knots). This claim can be proved using the argument we used in [2, Proposition 9.1]. For the convenience of the reader we provide details below.

Fix an analytic structure (X, p) on the cone over $Y_{\overline{n}}$ and note that the minimal resolution $\pi: \tilde{X} \rightarrow X$ provides a holomorphic filling (W, J) of $(Y_{\overline{n}}, \xi_{can})$. In particular, W is a regular neighborhood of the exceptional divisor $E = \bigcup E_j$ of π , the dual graph of which is just the circular plumbing graph $\Gamma_{\overline{n}}$. Since the curves E_j are holomorphic, by the adjunction formula, we have

$$(1) \quad \langle c_1(J), [E_j] \rangle = E_j \cdot E_j - 2 \text{ genus}(E_j) + 2 = E_j \cdot E_j + 2.$$

Now using a result of Bogomolov [3], deform the complex structure J so that (W, J') becomes a Stein surface, possibly after blowing down some (-1) -curves. Since W contains no topologically embedded spheres of self-intersection number -1 , (W, J') itself must be Stein. Note that (1) must continue to hold for J' even though the curves E_j are no longer holomorphic, since J and J' are homotopic to one another.

Now let $\{(W^i, J^i)\}$, for $i = 1, \dots, (n_1 - 1)(n_2 - 1) \cdots (n_k - 1)$, denote the finite set of Stein fillable contact structures on $Y_{\overline{n}}$ considered above by taking Legendrian realizations of the diagram in Figure 10. Denote by U_j^i a component of the corresponding Legendrian link and let S_j^i denote the associated surface in the Stein filling (W^i, J^i) obtained by pushing a Seifert surface for U_j^i into the 4-ball union 1-handle and capping off by the core of the corresponding 2-handle (see [10]). Notice that each W^i is diffeomorphic to W by a diffeomorphism which carries S_j^i to E_j for each j (see [11]).

Now, using the well-known identities

$$S_j^i \cdot S_j^i = \text{tb}(U_j^i) - 1, \quad \langle c_1(J^i), [S_j^i] \rangle = \text{rot}(U_j^i)$$

(see [10] for the second), observe that $\langle c_1(J^i), [S_j^i] \rangle = S_j^i \cdot S_j^i + 2$ precisely when $\text{rot}(U_j^i) = \text{tb}(U_j^i) + 1$. Since the latter equality holds exactly when all the cusps of U_j^i except one are up cusps, it follows that $\langle c_1(J), [E_j] \rangle = \langle c_1(J^i), [S_j^i] \rangle$ for each j precisely when all the extra zigzags are chosen so that the additional cusps are all up cusps, that is, when all the extra zigzags are chosen on the same fixed side (which is determined by the orientation of the Legendrian unknots). The proof is now completed by appealing to Lisca–Matić [15] and noting that in the finite list of Stein fillable contact structures on $Y_{\overline{n}}$ there is only one such Stein fillable contact structure up to isomorphism. \square

By a direct calculation one can check that the euler class of the contact structure on the boundary of the Stein surface given in Figure 9 vanishes, although this fact follows from

Lemma 8 and Proposition 9. Indeed, let $n_0 = 0$ and let $\tilde{\mu}_i$ denote the first homology class represented by a meridional circle to the curve with framing $-n_i$ for $i = 0, 1, \dots, k$ in Figure 10. Then $H_1(Y_{\overline{n}}; \mathbb{Z})$ is generated by the classes $\tilde{\mu}_0, \tilde{\mu}_1, \dots, \tilde{\mu}_k$ subject to following relations:

$$\begin{aligned} -n_1\tilde{\mu}_1 + \tilde{\mu}_2 + \tilde{\mu}_k &= 0 \\ -n_2\tilde{\mu}_2 + \tilde{\mu}_1 + \tilde{\mu}_3 &= 0 \\ -n_3\tilde{\mu}_3 + \tilde{\mu}_2 + \tilde{\mu}_4 &= 0 \\ &\dots \\ -n_k\tilde{\mu}_k + \tilde{\mu}_{k-1} + \tilde{\mu}_1 &= 0 \end{aligned}$$

By adding these up we get

$$(-n_1 + 2)\tilde{\mu}_1 + (-n_2 + 2)\tilde{\mu}_2 + \dots + (-n_k + 2)\tilde{\mu}_k = 0 \in H_1(Y_{\overline{n}}; \mathbb{Z}),$$

which implies that Poincaré dual of the euler class of the contact structure on the boundary of the Stein surface given in Figure 9 vanishes.

Notice that we can trade the 1-handle in the handlebody diagram depicted in Figure 9 with a contact $(+1)$ -surgery along a Legendrian unknot as described in [6] and obtain the contact surgery diagram of ξ_{can} on $Y_{\overline{n}}$ as shown in Figure 11.

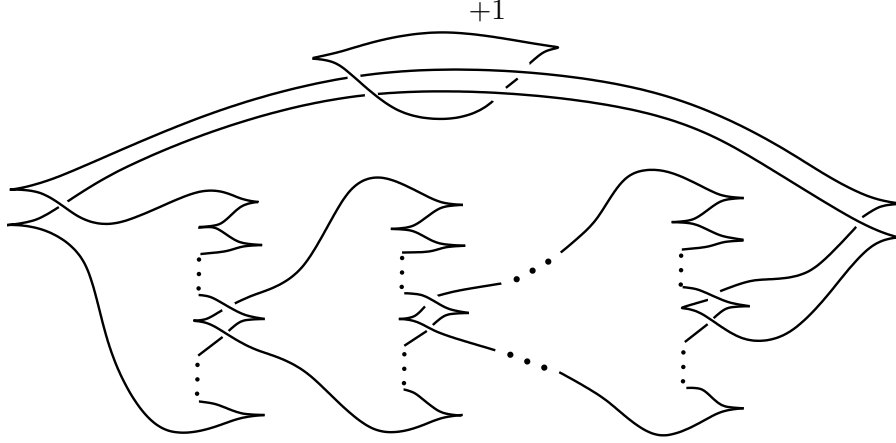


FIGURE 11. A contact surgery diagram for the canonical contact structure ξ_{can} on $Y_{\overline{n}}$.

Theorem 10. *The Euler class of the canonical contact structure on any singularity link which fibers over the circle vanishes.*

Proof. This is obtained by combining Lemma 1 with Lemma 8. □

We immediately get the following

Corollary 11. *A singularity link does not fiber the circle if the Euler class of its canonical contact structure is non-vanishing.*

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